Effect of AC electric field on thermal convective instability in a dielectric fluid saturated porous medium

Deepa K Nair, Potluri Geetha Vani and I. S. Shivakumara

Abstract— The effect of vertical AC electric field on the onset of convection in a horizontal layer of a dielectric fluid saturated Brinkman porous medium heated from below is investigated. The lower and upper boundaries of the fluid are considered to be rigid and isothermal. As observed in the classical dielectric fluid layer, oscillatory convection is found to occur provided the Prandtl number is less than unity. Therefore it is not a preferred mode of instability because the Prandtl number is greater than unity for dielectric fluids for poorly conducting liquids, so only the steady onset is considered for all the three types of boundary conditions. The eigenvalue problem is solved numerically using the Galerkin method. The effects of Darcy number, the ratio of viscosities, are analyzed on the stability of the system. The necessary conditions for the oscillatory instability to occur are independent of AC electric Rayleigh number. The effect of increasing AC electric Rayleigh number is to enhance the heat transfer and to hasten the onset of convection. The rigid boundaries offer more stabilizing effect on the system than free-free and rigid-free boundaries. Thus the foregoing study throws light on the control of electrohydrodynamic instability by a proper choice of velocity boundary conditions and AC electric field.

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Index Terms—AC electric field,dielectric fluid,porous medium,convection, isothermal boundaries, Darcy number, Rayleigh number.

1. Introduction

In view of understanding possible control of convection in liquid dielectrics and a control of heat and mass transfer in high-voltage devices by electric field, several studies have been carried out in the past to assess the effect of AC or DC electric field on natural convection in a horizontal dielectric fluid layer. Buoyancy driven convection in a fluid-saturated porous medium has been a subject of considerable interest amongst researchers because of its natural occurrence and also in many applications such as drying processes, thermal insulation, radioactive waste management, transpiration cooling, geophysical systems, and contaminant transport in groundwater, ceramic processing, solid matrix compact exchangers to mention a few. The developments which have been taken place in this field over the years are well documented by Nield and Bejan (1).

The effect of rotation on the onset of thermal convection in a horizontal fluid layer is well known for ordinary viscous fluids. Palm and Tyvand have (4) studied the linear stability problem of thermal convection in a rotating porous layer. Using Brinkman model, Jou and Liaw (5) have studied thermal convection in aporous medium subject to transient heating rotation. Qin and Kaloni (6) have studied the nonlinear stability of rotating porous layer by including the convective inertia term in the Brinkman model and they have shown that the effect of permeability is to stabilize the system. Recently, Shivakumara et al. (2) have discussed in detail the effect of vertical AC electric field on the onset of electrthermalconvection in a layer of dielectric fluid-saturated Darcy- Brinkman porous medium for various types of velocity boundary conditions, while the effect of Coriolis force due to rotation on such an instability problem has been investigated by Shivakumara et al. (3).

The intent of the present study is to investigate the effect of AC electric field on the criterion for the onset of electrothermalconvection in a dielectric fluid saturated Brinkman porous layer. In the present study three different types of boundary conditions are considered, namely (i) rigid–rigid (ii) free-free and (iii) lower rigid and upper free. The similarities and difference between these types of boundaries on the stability characteristics of the system are highlighted. The eigenvalue problem is solved exactly for free-free isothermal boundaries, while numerically using the Galerkin method for other boundary conditions.

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2. Mathematical formulation

We consider an infinite horizontal layer of dielectric fluid saturated porous medium of thickness d. The lower surface at $z=0$, and the upper surface at $z=d$ are maintained at constant temperatures $T_0+\frac{\Delta T}{2}$ and $T_0-\frac{\Delta T}{2}$, respectively. In addition, a vertical AC electric field is also imposed across the layer; the lower and upper surfaces are kept at an alternating potential $V_0-\frac{\Delta V}{2}$ and, $V_0+\frac{\Delta V}{2}$ respectively. The relevant basic equations under the Boussinesq approximations are:

$$\nabla\cdot\vec{q}=0 \qquad (1)$$

$$\frac{\rho_0}{\phi_p}\frac{\partial\vec{q}}{\partial t}+\frac{\rho_0}{\phi_p{}^2}(\vec{q}\cdot\nabla)\vec{q}=\left[-\nabla p+\rho\vec{g}+-\frac{1}{2}\vec{E}\cdot\vec{E}\nabla\varepsilon+\frac{1}{2}\nabla\left(\rho\frac{\partial\varepsilon}{\partial\rho}\vec{E}\cdot\vec{E}\right)\right]+\left[\tilde{\eta}\nabla^2\vec{q}-\frac{\eta}{\kappa}\vec{q}\right]$$

(2)

$$A\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\kappa\nabla^2T \qquad (3)$$

$$\rho=\rho_0\left(1-\alpha\left(T-T_0\right)\right) \qquad (4)$$

Since there is no free charge, the relevant Maxwell equations are

$\nabla\times\vec{E}=0$, $\nabla.\left(\varepsilon\vec{E}\right)=0$ (5a,b)In view of (5.5a), $\vec{E}$ can be expressed as $\vec{E}=-\nabla V$ (6)

Where V is the root mean square value of the electric potential. The dielectric constant is assumed to be a linear function of temperature in the form

$$\varepsilon=\varepsilon_0\left(1-\gamma\left(T-T_0\right)\right). \qquad (7)$$

Where γ (>0) is the thermal expansion coefficient of dielectric constant and is assumed to be small.

To study the stability of the basic state, we superimpose infinitesimally small perturbations on the basic state in the form

$$\vec{q}=\vec{q}',\ P=P_b+P',\ \vec{E}=\vec{E}_b+\vec{E}',\ T=T_b+T',\ \rho=\rho_b+\rho',\varepsilon=\varepsilon_b+\varepsilon' \qquad (8)$$

Where $\vec{q}'$, P', $\vec{E}$, T', ρ' & ε' are the perturbed quantities over their equilibrium counterparts. Substituting (8) into (1)-(7), linearizing the equations, eliminating the pressure from the momentum equation by operating curl twice and retaining the vertical component and non-dimensionalizing the resulting equations and performing linear stability analysis we get

$$\left[\frac{\omega}{Pr}\left(D^2-a^2\right)W+R_ta^2\Theta+R_{ea}a^2\Theta-R_{ea}a^2D\Phi\right]=\left[\Lambda\left(D^2-a^2\right)^2W-Da^{-1}\left(D^2-a^2\right)W\right]$$

$$\left[A\omega-\left(D^2-a^2\right)\right]\Theta=W \qquad (9,10)$$

$$\left(D^2-a^2\right)\Phi=D\Theta \qquad (11)$$

The boundary conditions are
$W=D^2W=\Theta=D\Phi=0$ (12)
on the stress-free boundary and
$W=DW=\Theta=\Phi=0$ (13)
on the rigid boundary.

3. Both boundaries free isothermal

Let us assume the solution in the following form such that they satisfy the respective boundary conditions:

$$W=A_1\sin(\pi z),\ \Theta=A_2\sin(\pi z),\ \Phi=A_3\cos(\pi z). \qquad (14)$$

where A_1-A_3 are constants. Substituting (14) into (9) - (13), we find the condition for the existence of a non-trivial eigenvalue is

$$\begin{vmatrix}\left[\Lambda\delta^4+Da^{-1}\delta^2\right]+\left(\frac{\omega\delta^2}{\mathrm{Pr}}\right) & -(R_t+R_e)a^2 & -R_{ea}a^2\pi\\ -1 & A\omega+\delta^2 & 0\\ 0 & \pi & \delta^2\end{vmatrix}=0 \qquad (15)$$

Expanding the above determinant yields an expression for the thermal Rayleigh number in the form

$$R_t=\frac{\delta^4\left(\delta^2+A\omega\right)}{a^4}\left\{\Lambda\delta^2+\frac{\omega}{\mathrm{Pr}}+Da^{-1}\right\}-\frac{R_t\delta^2}{a^2} \qquad (16)$$

To examine the stability of the system, the real part of ω is set to zero and take $\omega=i\omega$ in (16) we get

$$R_t=\frac{G\delta^6}{a^2}-\frac{1}{a^4}\left[\frac{A\delta^2}{\mathrm{Pr}}\right]\omega^2-\frac{a^2}{\delta^2}R_{ea}+i\omega_iN \qquad (17)$$

where $G=\Lambda\delta^4+Da^{-1}\delta^2$

$$N=AG\frac{\delta^4}{a^4}+\frac{\delta^6}{a^4\,\mathrm{Pr}} \qquad (18)$$

Since R_t is a physical quantity, it must be real and hence either $\omega_i=0$ or $N=0$ in (17).

4. Stationary convection

Enforcement of $\omega_i=0$ in (17) gives the condition for the occurrence of stationary convection. The corresponding expression for R_t is

$$R_t=\frac{\delta^4\left(\Lambda\delta^2+Da^{-1}\right)}{a^2}-\frac{a^2}{\delta^2}R_{ea}. \qquad (19)$$

To find the critical value of R_t , (19) is differentiated with respect to a^2 and equated to zero to get a polynomial in (a_c^2) in the form

$$7\Lambda\left[\left(a_c^2\right)^6+6\pi^2\left(a_c^2\right)^5+15\pi^4\left(a_c^2\right)^4+20\pi^6\left(a_c^2\right)^3+15\pi^8\left(a_c^2\right)^2\right]+\left[42\Lambda\pi^{10}-2R_{ea}\right]\left(a_c^2\right)+7\Lambda\pi^{12}=0 \qquad (20)$$

It is observed that the critical wave number varies with R_{ea},L and Da^{-1}. It is interesting to check (17) and (19) for the existing results in the literature under some limiting cases. Equations (17) and (19) coincide with the expressions given Lapwood in the case of Newtonian fluid through a porous medium

and is given by

$$R_t=\frac{Da^{-1}\left(\pi^2+a^2\right)^2}{a^2} \qquad (21)$$

We note that R_t attains its critical value R_{tc} at $a=a_c$ where

$$R_{tc}=a\pi^2Da^{-1}$$

and

$$a_c=\pi$$

5. Oscillatory convection

The onset of oscillatory convection corresponds to $N=0(\omega_i\neq 0)$ in (17). which gives Prandtl number less than unity. Therefore it is not a preferred mode of instability because the Prandtl number is greater than unity for dielectric fluids (for example, $Pr=480$ for corn oil, 100 for silicone oil and 10000 for caster oil).

5.1 Both boundaries rigid

Since the occurrence of oscillatory convection is not a preferred mode of instability for dielectric fluids, we set ω = 0 in (9) - (11). As in the case of free isothermal boundaries, an exact solution is not possible for the rigid boundaries and the Galerkin method is adopted to solve the resulting eigenvalue problem. Accordingly, the variables are written in a series of basis functions as

$$W=\sum A_iW_i\,,\ \Theta=\sum B_i\Theta_i\,,\,,\ \Phi=\sum C_i\Phi_i \qquad (22)$$

where A_i, B_i and C_i are constants. The basis functions and Φ_i will be represented by the power series satisfying the boundary conditions. Substituting (22) into (9)–(11) (after noting ω = 0), multiplying the resulting momentum equation by $W_j(z)$,energy equation by $\Theta_j(z)$, electric potential equation by $\Phi_j(z)$; performing the integration by parts with respect to z between z = 0 and z = 1 and using the boundary conditions, leads to the following system of linear homogeneous algebraic equations:

$$E_{ji}A_i+F_{ji}B_i+G_{ji}C_i=0$$

$$H_{ji}A_i+I_{ji}B_i\ =0 \qquad (23)$$

$$J_{ji}B_i+K_{ji}D_i=0$$

where, (5.33)

$$E_{ji}=<\Lambda D^2W_jD^2W_i+2a^2DW_jDW_i+a^4W_jW_i>$$
$$+Da^{-1}\langle DW_j,DW_i\rangle+a^2\langle W_j,W_i\rangle\ \ F_{ji}=-<R_t\,a^2W_j\Theta_i+R_ea^2W_j\Theta_i>,$$
$$G_{ji}=<R_{ea}a^2W_jD\Phi_i>,\ \ H_{ji}=<\Theta_jW_i>$$
$$I_{ji}=-<D\Theta_jD\Theta_i+a^2\Theta_j\Theta_i>,$$
$$J_{ji}=-<\Phi_jD\Theta_i>,\ \ K_{ji}=-<D\Phi_jD\Phi_i+a^2\Phi_j\Phi_i>$$

Here the inner product is defined as $<f_1f_2>=\int_0^1 f_1f_2\,dz$.

(24)

The following three types of velocity boundary combinations are considered foepr discussion:

(i) both boundaries rigid (5.35)

$W=DW=\Theta=D\Phi=0$ at $z=0,1$ (25)

(ii) both boundaries free

$W=D^2W=\Theta=D\Phi=0$ at $z=0,1$ (5.36) (26)

(iii) lower boundary rigid and upper boundary free

$W=DW=\Theta=\Phi=0$ at $z=0$

$W=D^2W=\Theta=D\Phi=0$ at $z=1$ (27)

The boundary conditions allow us to choose the trial functions as follows

(i) Rigid –rigid boundaries

$$W_i=z^{i+1}-2z^{i+2}+z^{i+3}$$
$$\Theta_i=z^i-z^{i+1}$$
$$Z_i=z^i-z^{i+1} \qquad (28)$$

(ii) Free-free boundaries

$$W_i=z^{i+3}-2z^{i+2}+z^i$$
$$\Theta_i=z^i-z^{i+1} \qquad (29)$$
$$Z_i=z^{i+1}-2z^{i+2}+z^{i+3}$$

(iii)Lower rigid and upper free boundaries

$$W_i=2z^{i+3}+3z^{i+1}-5z^{i+2}$$
$$\Theta_i=z^i-z^{i+1} \qquad (5.39)\ (30)$$
$$Z_i=z^{i+1}-2z^{i+2}+z^{i+3}$$

It may be noted that the above polynomial trial functions automatically satisfy the respective boundary conditions.

The thermal Rayleigh number R_t or the AC electric Rayleigh number R_{eac} is taken as the eigenvalue. The critical values of R_t or R_{ea} as the case may be are computed as a function of wave number a for assigned values of other parameters. The results presented here are for $i=j=5$ the order at which the convergence is achieved, in general.

6. Results and Discussion

The effect of vertical AC electric field on the criterion for the onset of thermal convection in a layer of dielectric fluid is investigated. The bounding surfaces of the fluid layer are considered to be either rigid-rigid or free-free or lower rigid and upper one free. The analytical study carried out for isothermal free-free boundaries reveals that the necessary conditions for the occurrence of oscillatory convection are independent of vertical AC electric field. Since the Prandtl number is much greater than unity for dielectric fluids, occurring of oscillatory convection as a preferred mode of instability is discarded. Under the circumstances, the study has been restricted to stationary convection. For the remaining boundary combinations, the eigenvalue problem is solved numerically using the Galerkin technique.

Figure 1 displays the variation of critical thermal Rayleigh number R_{tc} as a function of R_{ea} when $\Lambda = 1$ and $Da^{-1} = 1$ for free-f convection. From the figure it is also seen that the rigid-rigid boundaries are more stabilizing compared to free-free boundaries due to the increased suppression of disturbances in the case of rigid boundaries. The variation of corresponding critical wave number a_c is shown in Figure 2. It is observed that, the critical wave number increases with increasing R_{ea} considerably in the case of free-free boundaries, while the variation is found to be insignificant in the case of rigid-rigid and rigid-free boundaries.

It is a known fact that the effect of fluid viscosity, in general, is to resist continued motion of the fluid by dissipating the kinetic energy of the fluid. In Figure 3 the variation of R_{tc} is exhibited as a function of R_{ea} for different values of Λ. It is observed that R_{tc} is an increasing function of Λ indicating its effect is to stabilize the fluid motion against electrothermalconvection. Thus the effect of increasing Λ is to delay the onset of electrothermalconvection. Besides, the effect of increasing Λ is to decrease the critical wave number and hence its effect is to increase the size of convection cells (see Fig.4). The effect of increase in Da^{-1} is to increase the critical thermal Rayleigh number and thus has a stabilizing effect on the system and the same is evident from Figure 5. In addition, the effect of increasing Da^{-1} is to increase the critical wave number initially, while an opposite trend is noted once the value of R_{ea} exceeds certain value (Fig. 6).

7. Conclusions

The results of the foregoing study may be summarized as follows. The effect of AC electric field on the onset of convection in a dielectric fluid saturated porous media is analyzed with the object of understanding control of electrohydrodynamic instability for rigid-rigid, rigid-free and free-free boundaries. Analytical expression for the occurrence of steady and oscillatory convection is obtained for isothermal free-free boundaries. The necessary conditions for the oscillatory instability to occur are independent of AC electric Rayleigh number. The effect of increasing AC electric Rayleigh number is to enhance the heat transfer and to hasten the onset of convection. The rigid boundaries offer more stabilizing effect on the system than free-free and rigid-free boundaries. Thus the foregoing study throws light on the control of electrohy drodynamic instability by a proper choice of velocity boundary conditions and AC electric field.

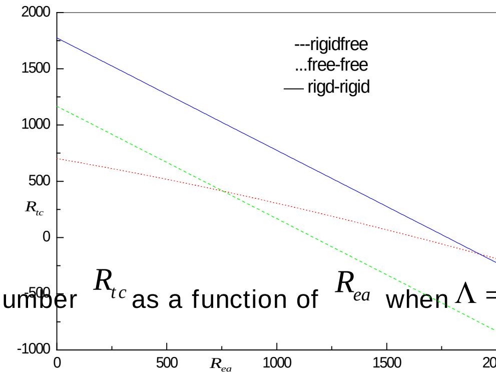

Fig 1: Variation of R_{tc} with R_{ea} when $Da^{-1} = 1, \Lambda = 1$ for free- free rigid-rigid & rigid-free boundaries.

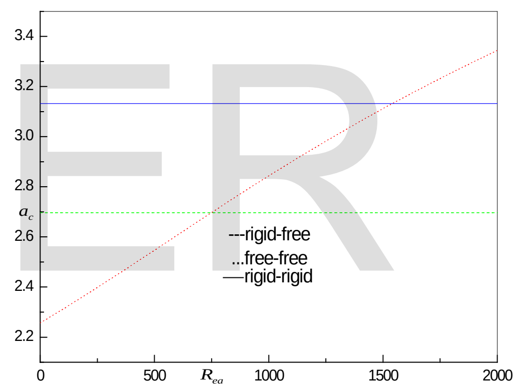

Fig 2: Variation of a_c with R_{ea} when $Da^{-1} = 1, \Lambda = 1$ for free- free rigid-rigid & rigid-free boundaries.

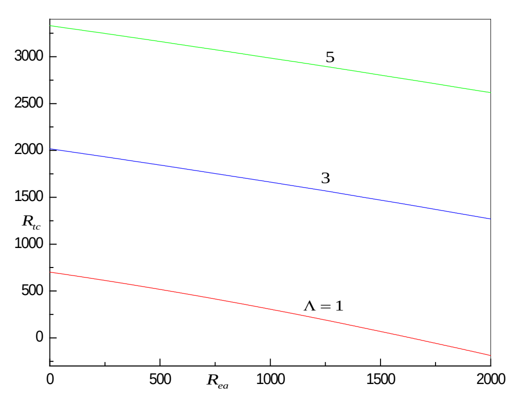

Fig 3: Variation of R_{tc} with R_{ea} when $Da^{-1} = 1, \Lambda = 1,3,5$ for free- free boundaries.

ISSN 2229-5518

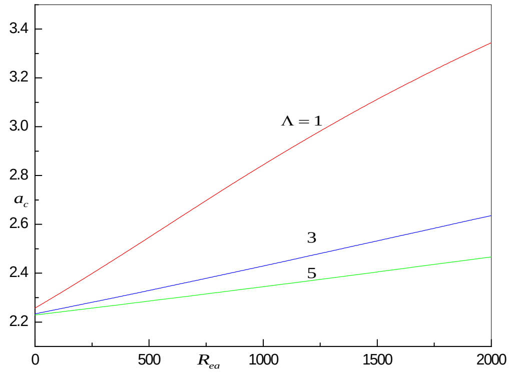

Fig 4: Variation of a_c with R_{ea} when $Da^{-1} = 1, \Lambda = 1,3,5$ for free- free boundaries.

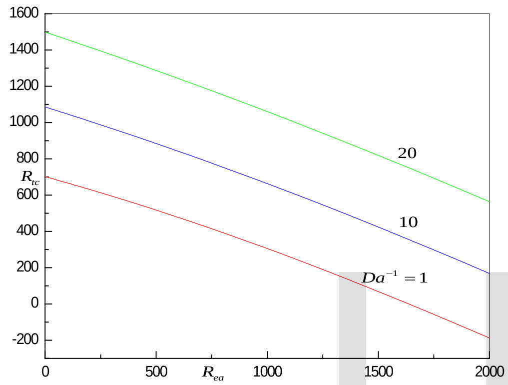

Fig 5: Variation of R_{tc} with R_{ea} when $Da^{-1} = 1,10,20, \Lambda = 1$ for free-free boundaries.

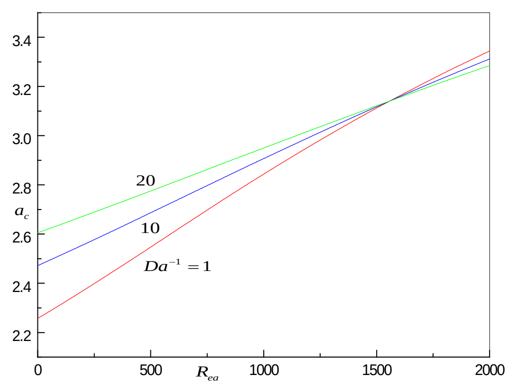

Fig 6: Variation of a_c with R_{ea} when $Da^{-1} = 1,10,20, \Lambda = 1$ for free-free boundaries.

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